Optimal Formation of Mobile Robots Transporting a 6-DoF Payload through Tight Spaces

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***Abstract*— This paper addresses the problem of identifying feasible optimal formations for a group of non-holonomic mobile robots manipulating a payload with 6-Degree Of Freedom(DoF) movement through narrow spaces. We solve this problem in two stages. Initially we find a feasible obstacle-free trajectory for the system. Subsequently, we arrive at optimal formation that efficiently moves through narrow spaces by simultaneously minimizing a kinetic energy metric and a geometric stability criterion. The best robot motion plans are devised through multi-objective optimization. We demonstrate that stable and energy efficient formations are achievable, independent of system dynamics for a multi-robot, *quasi-static*, payload transport system moving through narrow spaces.**

I. INTRODUCTION

Multi-robot payload transportation is an energy intensive task. The weight of a heavy payload is burdened by formation of non-holonomic mobile robots and therefore conservation of energy of these robots is an important consideration. When weight of a payload moving in 6-DoF is greater than the cumulative weight of the formation of robots and manipulators, transporting it through tight spaces can lead to many unstable formations, thereby causing system imbalance. This results in the need to estimate stability of system so that the system remains balanced.

Bhatt et. al. [1] developed instantaneous kinetic energy metric to optimize energy and plan motion of mobile manipulators that carry a payload having 3-DoF movement. The work did not consider stability of the formation as a criterion. Abbaspour et al. [2] considered dynamics of system to determine optimal formation of mobile robots to transport a 3-DoF payload along a given trajectory. The mobile robots used 1-DoF manipulators to transport a payload with 3-DoF. Jiao, Jile, et al. [5] address motion planning for a system of mobile manipulators and a 6-DoF payload through cluttered environments. The work did not consider optimality or stability of formation of robots as a criteria.

Motion planning and dynamic model evaluation of such a multi-mobile manipulator system is challenging in view of (a) temporal changes in position and orientation of payload, (b) formation of robots and (c) need for movement through tight spaces. Therefore, a simple estimate of the stable configuration has to be determined. Novel aspects of the work can be summarized as follows. Firstly, a guided Rapidly Exploring Random Tree(RRT) based offline 2-D motion planner to determine 6-DoF motion plan of the payload is devised. Secondly, an online motion planner is proposed that derives stable and energy efficient motion plans for the robots using multi-objective optimization algorithms.

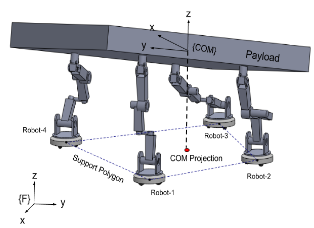

Fig. 1. System Setup: 4 differential drive mobile manipulators connected by support polygon, carrying a payload

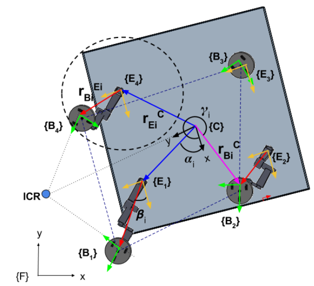

Fig. 2. Bottom View: C: Center of Mass Frame, E: End-effector Frame and B: Robot Frame. r_X^Y: Vector from frame Y to X. α_i, β_i and γ_i: Angle b/w different frames.

II. PAYLOAD MOTION PLANNING

The proposed multi-robot system under consideration, consists of a payload with uniform mass density and N mobile robots, each mounted with a 6-DoF manipulator as shown in Fig 1. Each robot has a circular feasible region defined w.r.t projection of the grasp point on the XY plane as shown around Robot frame $\{B4\}$ in Fig 2. The 3-D environment under consideration has many closely spaced columns of obstacles as shown in Fig 4. In view of this, a modified RRT algorithm, with *intermediate goals as guidance* is used. The tree is grown using unicycle kinematics to ensure kinematic feasibility of path. Instead of growing the tree on all 6-DoF, projection of the payload and the obstacles on XY plane are determined. The tree is now grown on 3-DoF for the Center of Mass(CoM) of payload projection in 2-D. The *Roll* and *Pitch* of the payload are mapped to *Length* and *Width* of the payload projection in XY plane. Obstacle avoiding heuristics at each node are used to determine length and width of payload projection as the tree grows and finally mapped to roll and pitch of the payload. The CoM of payload remains at constant height throughout the transportation. The motion planning methodology developed can be conservatively applied to a payload of any convex or non-convex shape by finding its *optimal 3-D bounding box*.

III. FORMATION OPTIMIZATION

As the payload moves along a given trajectory, regional constraints of the robot are varied dynamically to avoid obstacles and inter-robot collisions. During payload transportation, changes in formation result in variations in energy consumption as well as stability of the system. At every time step, in order to identify efficient, stable and feasible robot

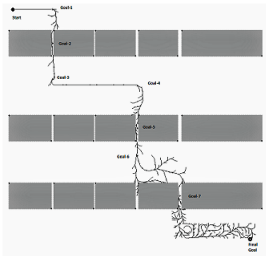

Fig. 3. Result of guided RRT for payload motion planning

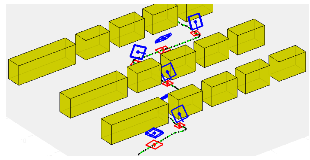

Fig. 4. Payload and its projection moving through tight spaces in 3-D

formations, we minimize metrics based on (a) Kinetic Energy (K.E.) and (b) Support Polygon [4] inspired stability metric. K.E. of a robot can be computed using the following equation,

$$E_i = \frac{1}{2}(m_i(\|\boldsymbol{v_i^C}\|)^2 + I_{zzi}(\|\boldsymbol{\omega_i^C}\|)^2) \tag{1}$$

where, m_i, I_{zzi} are the mass and mass moment of inertia and v_i^C, ω_i^C are the linear and angular velocities of the i^{th} robot. Energy E_t of formation of N robots at time instant t,

$$E_t = \sum_{i=1}^{N} E_i \tag{2}$$

If the projection of the CoM of payload onto ground lies within the polygon connecting the N robots then the system would remain statically stable [4]. Minimizing Euclidean distance between the centroid of the support polygon and the projection of the CoM of the payload on to ground provide good estimate of static stability. The stability metric at time instant t can be defined as,

$$S_t = \frac{1}{N}\sqrt{\Big(\sum_{i=1}^{N}\|\boldsymbol{r_{B_i}^C}\|\cos(\gamma_i)\Big)^2 + \Big(\sum_{i=1}^{N}\|\boldsymbol{r_{B_i}^C}\|\sin(\gamma_i)\Big)^2} \tag{3}$$

Individually, each of the the metrics leads to conflicting robot positions. In view of this, we solve this problem through multi-objective optimization. The multi-objective optimization problem can be stated as follows,

$$\begin{aligned} &\underset{X}{\text{Minimize}} \quad f_1 : E_t \quad f_2 : S_t \\ &\text{subject to,} \ \boldsymbol{X_l} \ < \mathbf{X} < \ \boldsymbol{X_u} \end{aligned} \tag{4}$$

where, $\boldsymbol{X}$ represents design variables which is a $1X2N$ matrix $\boldsymbol{X}$ given by Equation 5. The terms $\boldsymbol{X_l}$ and $\boldsymbol{X_u}$ in equation 4 represent the lower and upper bounds for $\boldsymbol{X}$.

$$\boldsymbol{X} = \begin{bmatrix} \|\boldsymbol{r}_{B_1}^{E_1}\| & \alpha_1 & \|\boldsymbol{r}_{B_2}^{E_2}\| & \alpha_2 & \cdots & \|\boldsymbol{r}_{B_N}^{E_N}\| & \alpha_N \end{bmatrix} \tag{5}$$

IV. Results and Discussion

The system consists of a payload of dimensions $1.5(m)$X$1.5(m)$ and four identical cylindrical robots of mass 10 Kg and radius 0.1 m. Maximum linear velocity of payload CoM is 2 ms^{-1}. The limit on $\|r_{B_i}^{E_i}\|$ is 0.7m. An obstacle free payload trajectory through tight spaces is obtained as shown in Fig. 3 and Fig. 4 by using the payload motion planning algorithm discussed in Section II.

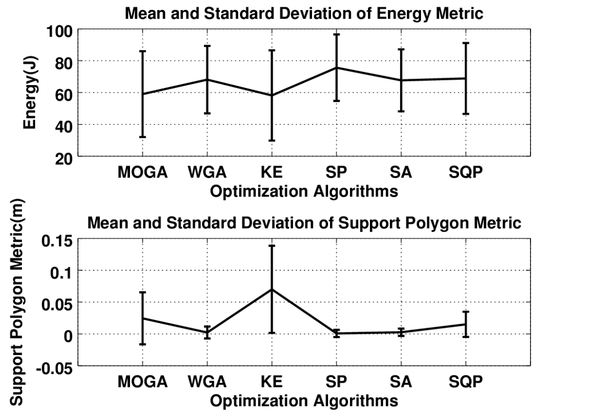

Fig. 5. Mean and Std. Deviation of (a) Instantaneous Energy Utilized by all Robots and (b) Stability Metric, **along the optimized trajectories**

To solve equation (4), optimization algorithms such as, Multi-Objective Genetic Algorithm(MOGA), Simulated Annealing(SA), Weighted Sum Genetic Algorithm(WGA) and Sequential Quadratic Programming(SQP) are examined and compared. In addition to the above, energy metric(KE) and stability metric(SP) are optimized independently and compared with the solutions of multi-objective optimization in order to verify their effectiveness.

It is observed that MOGA and WGA perform good in both mean and standard deviation for energy metric. WGA and SA perform the best for support polygon metric. WGA with equal weight for both objectives performs slightly better overall. It can be observed that optimizing kinetic energy gives better results for energy metric while stability is low and vice versa when support polygon alone is optimized. Larger the payload dimensions and limit on $\|r_{B_i}^{E_i}\|$, larger the value of the support polygon metric for KE, leading to unstable formations. This necessitates the use of multi-objective optimization to ensure efficient and stable formations. Thus, the proposed approach successfully ensures that the support polygon formed by the formation of robots does not drift away from the projection of center of mass of the payload, thereby ensuring a stable system. Simultaneously, by the use of an instantaneous kinetic energy metric the energy utilized by the system is minimized. Video of simulations and results can be found at `https://youtu.be/ZknDhmz4J-M`.

References

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